Modeling Fixed Rate MBS Prepayments

ABSTRACT

December, 2005

The Beyondbond Prepayment Model is a dynamic econometric model that applies economic theory to analyze mortgagor's prepayment behaviors and then fits them using historical US residential prepayment information. In order to develop the model, essential mortgage characteristics such as amortization term, age, contract rate, and principal factor as well as crucial economic variables such as, housing turnover rate, the prevailing market mortgage rate, and yield curve information over time were selected to determine the prepayment factors.

The refinance factor measures the refinancing incentive based on the cumulated in-the-money call options with the maturity equal to the loan term, and is considered to be the primary incentive for prepayments. The age factor discounts some of the prepayment impact for less seasoned loans. New mortgagors usually have less incentive to prepay initially due to the possibility of having to incur refinancing costs. The burnout factor, which is caused by the population heterogeneity, discounts the refinancing incentive via the remaining principal factor of the mortgages. A synthetic principal factor that only accumulates in-the-money prepayment options is introduced to more accurately capture the burnout.

While the model filters several important prepayment factors, other components such as the housing turnover rate, initial financing expense, and the issue year for the mortgage pool are exogenous non-neglected variables and need to be fine-tuned from time to time. In addition, the seasonality factor measures the impact on prepayment due to the seasonality over the year. The yield curve factor is used to measure cross maturities refinancing. The cash-out factor is the impact on prepayment driven by general housing appreciation. The equity factor reflects the refinancing rate at a lower mortgage rate due to improved collateral LTV level while the credit factor reflects the refinancing rate due to the improvement in the borrower's credit score. The model also considers curtailments and early payoffs which induce prepayments when the loan balances become small toward the end of the loan terms. Determinant factors for defaults such as loan-to-value ratio, debt-to-income ratio, and credit history are also analyzed.

All of the factors above were quantified to form our prepayment model. The model specification employs spline functions, especially cubic spline. Using different forms of splines, the model is able to filter out more crucial prepayment factors such as refinance, age, and burnout.

The applications of the model include prepayment forecast generation as well as price calculation, options adjusted spread evaluation, and scenario analysis for mortgages and mortgage-backed securities.

This paper only focuses on the fixed rate residential mortgages. Other types of prepayment modeling will reserved for another paper for detailed discussion.

C.H.Ted Hong
CEO & President
646.313.3330
ted@beyondbond.com

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I. INTRODUCTION

Modeling prepayment behavior is an ongoing research project. The importance of prepayment analysis has been recognized since the start of the mortgage market because mortgage prepayments can severely affect the price performance of the underlying mortgages. It became more important as the Mortgage-Backed Securities (MBS) market developed over the past two decades to become the fastest growing and largest sector in the US capital markets. High liquidity and relatively strong performance of the MBS products have attracted a wide range of investors who have demanded a better understanding of their investments. The need for this understanding is especially acute when dealing with mortgage derivatives and structured products that can be highly sensitive to prepayments and require advanced pricing technique such as options adjusted refinancing.

The current interest rate environment, the lowest in forty years, has been a key determinant in causing mortgage prepayments. It has triggered a new wave of refinance and pushed the prepayment trend to new highs. Mortgage research obviously requires sophisticated analytical tools with prepayment modeling capability to deal with such situations. A collection of various types of the mortgage prepayment models that can provide constant updates and feedbacks with respect to the market momentum becomes even more important for the investors.

Several steps take place to develop a prepayment model. Mining the historical data to gather basic summary statistics is usually the first step in building a prepayment model. Then, an economic model with a functional form will help to understand the prepayment phenomenon over time. Empirically fitting the historical prepayment data certainly would add further research creditability for the model. Finally, systematically projecting prepayment rates in a consistent basis would be useful and practical for the market participants wanting to utilize the model. However, if any up-to-date news was not originally considered as the model parameters were being fitted, the ability to fine tune the model coefficients will help to incorporate the most up-to-date feedbacks and previously missed observations.

Once the model is developed, only the empirical use of the results based on the prepayment model can support and lend certain creditability to the model.

The rest of paper is arranged as follows. The Section II describes the prepayment data in summary. The Section III examines the components of the Beyondbond Prepayment Model on an individual basis and provides a theoretic framework and its detailed model specification. The Section IV presents some of the empirical results and their numerical illustration. Finally, it concludes with certain findings, parameters updating procedures and areas of further research.

II. DATA

We selected the US residential mortgage data based on three agencies, Federal Home Loan Mortgage Corporation (FHLMC), Federal National Mortgage Association (FNMA), and Government National Mortgage Association (GNMA). The data, updated monthly, are historical prepayment rates on a pool basis from January 1984 to today. The data was compiled in Single Monthly Mortality (SMM) rates over the period in a cohort data format. The cohort prepayment data were categorized by their originating agencies and amortization types. Within each category, each mortgage pool contains Gross Weighted Average Maturity (WAM), Gross Weighted Average Contract Rate (WAC), Net Coupon Rate, Original Balance, Outstanding Balance, Principal Factor, Weighted Average Pool Age (AGE), and Prepayment Rate in a SMM format.

The basic statistics are shown as Figure 1. The average values were all weighted by their outstanding balances. In general, FNMA's prepayments were faster than GNMA's and the data were very dispersed when compared by their standard deviations around their mean values respectively.[1]

The outstanding pool balances being used here as the weights are utilized again in later sections. More detailed explanation is provided in our empirical results section.

Figure 1.

Agency/Maturity	Average Age (month)	Average CPR (%)	Standard Deviation of CPR
FNMA 30-year	37	24.52	23.36
GNMA 30-year	53	17.55	17.20
FNMA 15-year	24	19.17	19.71
GNMA 15-year	42	14.35	13.31
FNMA 7-year Balloon	12	24.52	25.00

[1]The outstanding pool balances being used here as the weights are utilized again in later sections. More detailed explanation is provided in our empirical results section.

III. THEORETIC FRAMEWORK AND ITS COMPONENTS

What Causes Prepayments

‣ Turnover
‣ Refinance
‣ Curtailment and early payoff
‣ Default

Figure 2.   **History of Fannie Mae 7s 1987 Issue**
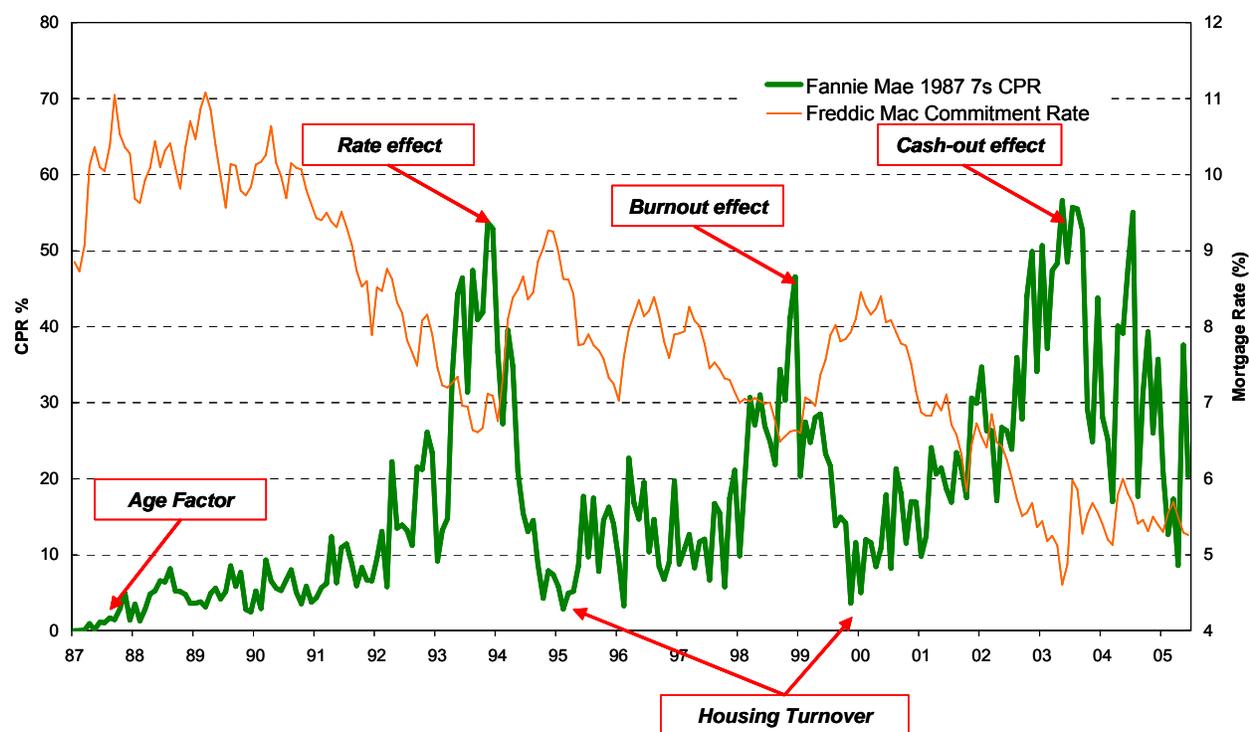
Sources: Fannie Mae, Freddie Mac, and Beyondbond

Essentially, prepayment is an option that a mortgagor/borrower has to prepay the remaining balance of one's mortgage in full or in partial amount at any time prior to its stated maturity without penalty. It is an American option for a mortgagor who can exercise the option whenever the mortgage market value is higher than the remaining balance. If exercising the option by refinancing, the mortgagors will have an incentive to whenever the prevailing refinancing rate is lower than their contract rates. The likelihood of mortgagors to exercise the option also depends on their refinancing incentives, costs, credit abilities, and mobility.

Comparing the historical prepayment data between premium and discount coupon mortgages, given the same age assumption, studies can support the above conclusion. This is neither easy nor obvious, however, when the comparison is made among only the premium coupon mortgages. In addition, other characteristics of the underlying mortgage and the economic environment over time can also influence the prepayment decision.

Prepayment Components

In our prepayment framework, we considered nine major factors as followings:

- ▸ Housing Turnover Rate
- ▸ Seasonality
- ▸ Cash-out
- ▸ Refinance
- ▸ Age
- ▸ Burnout
- ▸ Yield Curve
- ▸ Equity
- ▸ Credit

The prepayment incentive, which stems from the refinancing portion and is measured by the difference between the mortgage rate and its prevailing refinancing rate, is commonly recognized as the refinance factor. The initial origination fee and the loan closing expenses usually take a few years to depreciate, and that discourages the new mortgagors from prepaying their mortgages at the start of the mortgage term. This contributes to the age factor. The mobility of mortgagors is affected by the timing of housing construction, school year, and weather considerations, which can be considered as factors of seasonality. The heterogeneity of refinancing population, causing mortgagors to respond with different prepayment incentive reactions to the same refinancing rate, can be filtered out as the burnout. The cross maturities refinancing incentive that relates to yield curve shape can be measured by the yield curve factor.

Mortgage Agencies and Amortization Types

Since the mortgage agencies have their own criteria to underwrite and pool their mortgages, their prepayment behaviors are not necessarily the same. For example, the GNMA mortgage pools, which are either insured by FHA or guaranteed by VA loans, qualify only low income households and veterans. Their prepayments can behave quite different from the conventional loans which are originated by FNMA or FHLMC. The amortization types based on diverse loan amortization schedules and maturity terms such as 30-year, 15-year, 7-year balloon, and 5-year balloon have different prepayment rates. Different payment types such as fixed rate, adjustable rate, or GPM can also result in different types of prepayments. Their prepayment structures can be demographically different and should be estimated separately. This paper only focuses on the fixed rate residential mortgage. Other types of prepayment modeling will reserved for another paper for detailed discussion.
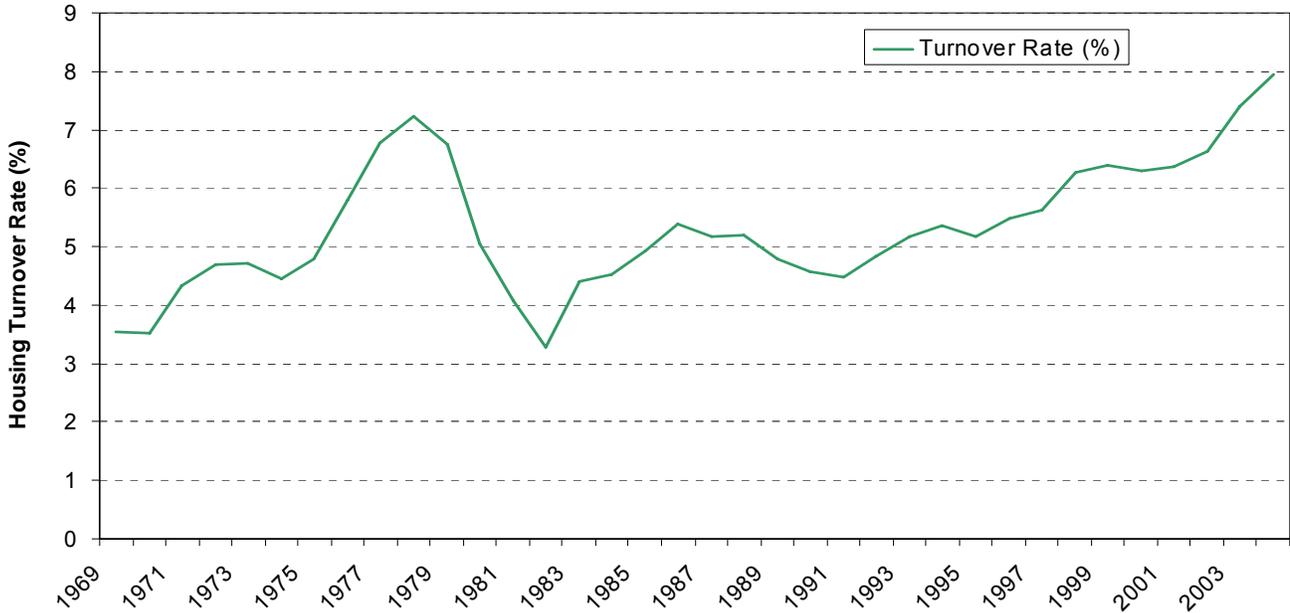
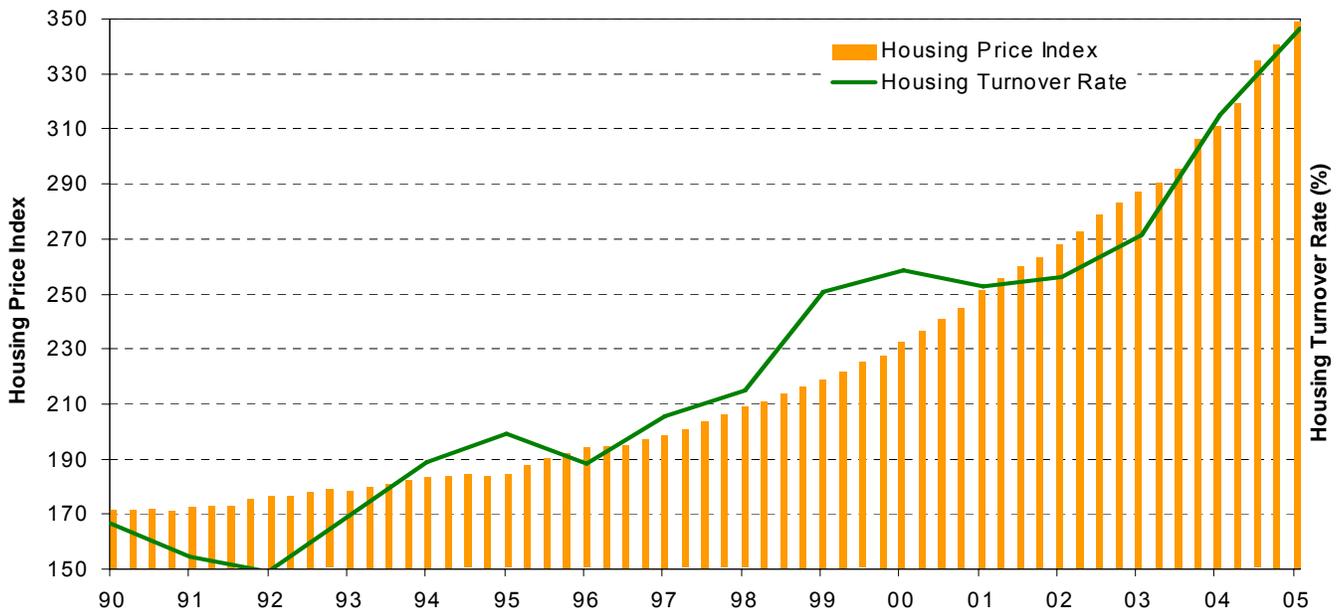## Housing Turnover Rate

Prepayment based on long-term housing turn-over rate that is composed of existing sales over single-family owner's housing stock.

Figure 3.   **Housing Turnover Rate 1969 - 2004**

Sources: National Association of Realtors and Beyondbond

Figure 4.   **Housing Price Appreciation vs. Turnover Rate**

Sources: National Association of Realtors, Bloomberg, and Beyondbond

Seasonality

Monthly seasonality is generally believed to affect prepayments. The belief stems from the mobility of mortgagors, time of housing construction, school year, and weather considerations. For a specific month of the year and ceteris paribus, prepayment rates are directly affected by the related month-of-year's coefficient. Usually, the seasonality pattern tends to be more active in the spring, rises to peak in the summer, decreases through the fall, and slows down even more in the winter. The pattern may be different geographically and demographically. Figure 5 shows the estimated seasonality pattern for FNMA 30-year MBS.

Figure 5.   **FNMA 30-Year Seasonality**
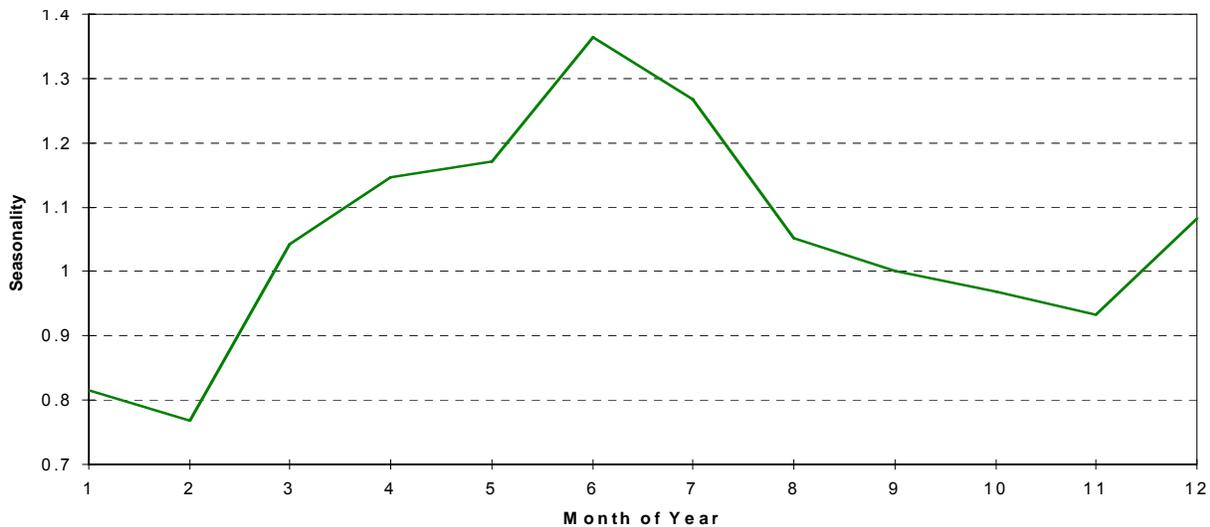
Sources: Beyondbond

Prepayment due to home sale cycle seasonality.

Figure 6.   **Existing Home Sales, 1995 - 2005**
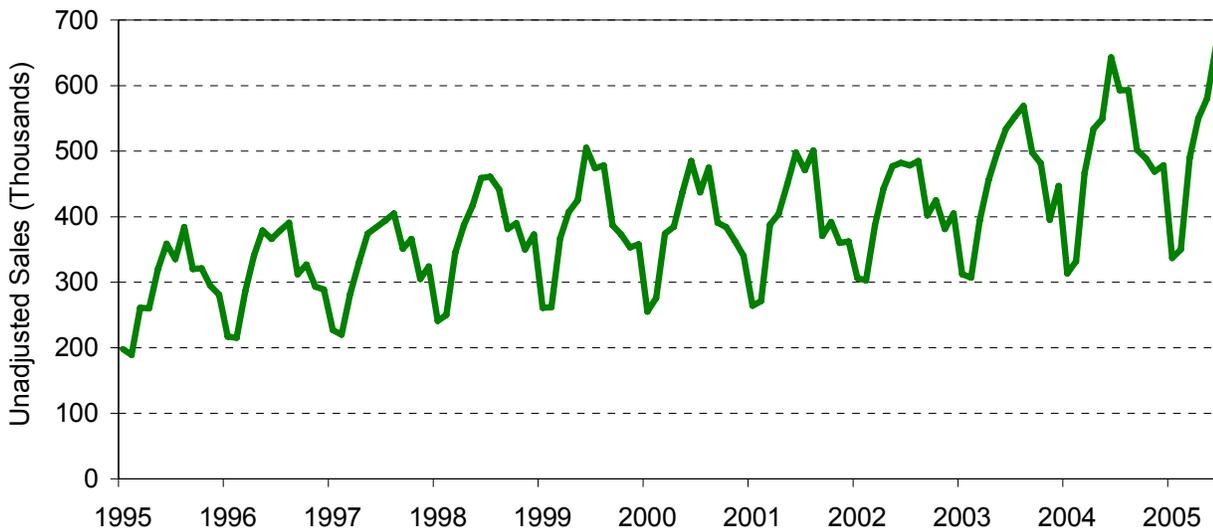
Sources: Bloomberg and Beyondbond

Cash-out

Prepayment driven by general housing price appreciation.

Figure 7.   **Cash - Out Effect (U.S. Housing Price Index)**
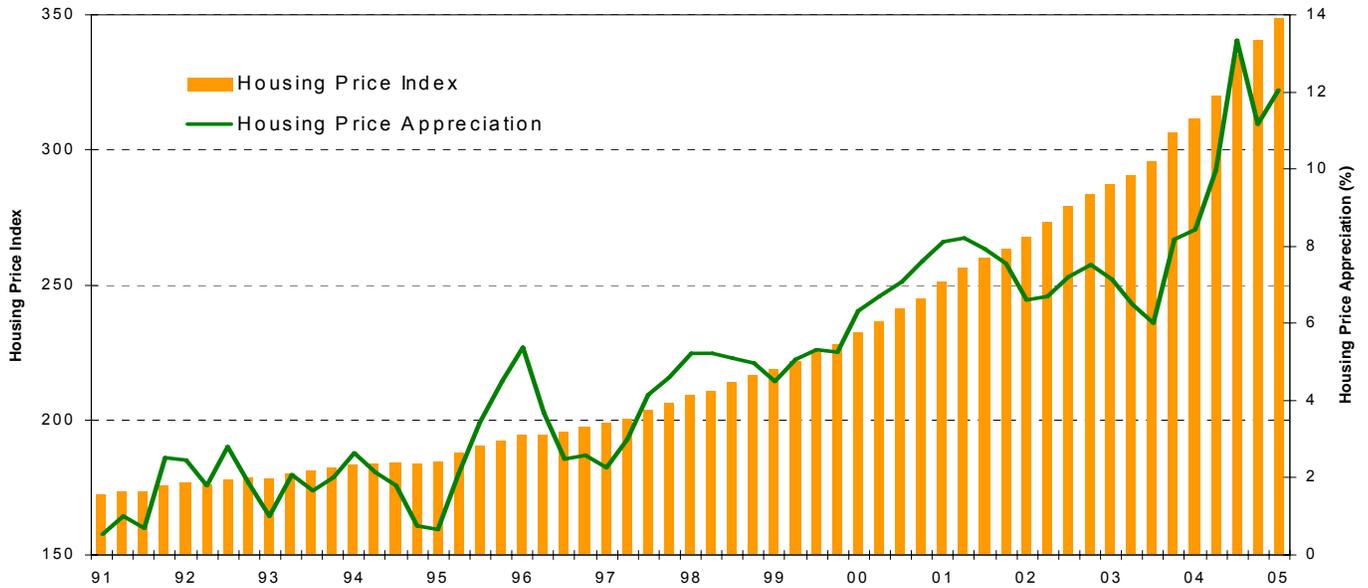
Sources: Bloomberg and Beyondbond

Figure 8.   **Housing Price Appreciation vs. MBA REFI Index**

Sources: Bloomberg and Beyondbond

Figure 9.   **1986 Fannie Mae 8.5s and 9.0s**
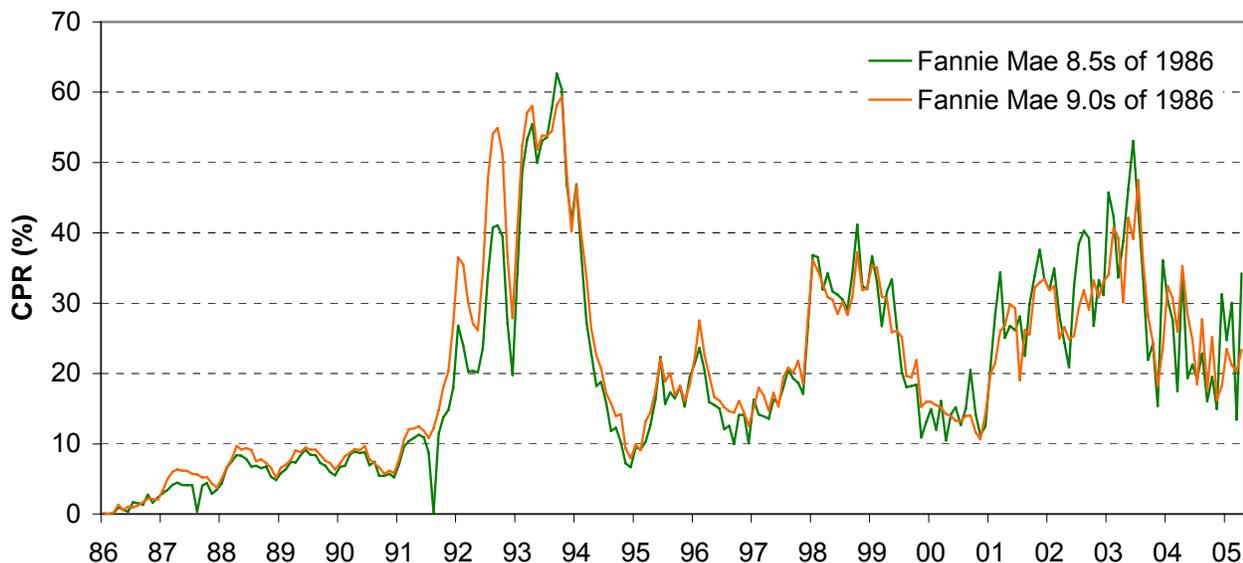
Sources: Bloomberg

Refinancing Incentive

It is well known that mortgagors tend to prepay whenever their mortgage rate is greater than the prevailing refinancing rate.[2] In other words, mortgagors tend to prepay while their prepayment option is in-the-money. The refinancing costs such as origination points, processing expenses, title insurance, and loan qualification difficulties, however, determine whether the option is worth exercising or not. Since a drop in interest rates increases the value of the prepayment option, prepayments speed up as interest rates decrease. Due to processing time for loan originations, lagged interest rates are more relevant to prepayments.

If the averaged conventional commitment rates of the primary mortgage market surveyed by Freddie Mac is assumed to be the refinancing incentive threshold (current coupon mortgage rate) and if the refinancing incentive is assumed to be reflected by the difference between WAC and FRM30Y, then an S-shaped refinancing pattern can be drawn based on historical data. Figure 10 displays FNMA 30-year CPR versus refinancing incentive for three age brackets based on actual prepayments from January, 1990 to October, 1993. The brackets "New", "Moderately Seasoned", and "Seasoned" denote mortgage ages of less than 30 months, between 30 and 60 months, and greater than 60 months, respectively. The lower bound for seasoned MBS's CPR can be thought of as the average housing turnover rate, which is approximately 6%.

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[2]The prevailing interest rate is the threshold for exercising the prepayment option and can be thought of as the current coupon mortgage rate. Any greater/lower contract rate mortgage is considered to be a premium/discount coupon mortgage. Note that the prevailing mortgage rate can be greater than the actual FNMA current coupon rate due to heterogeneous refinancing costs.
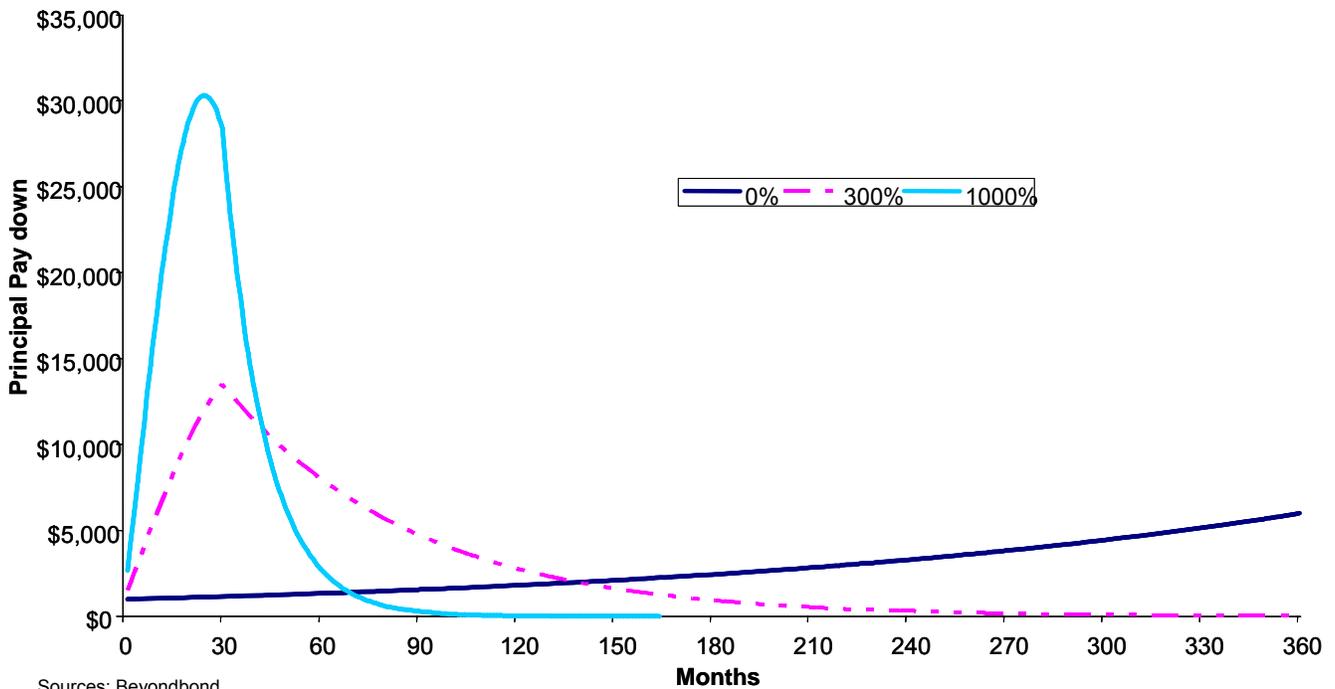
Figure 10.   **Prepayments of FNMA 30-Year Conventional MBS**
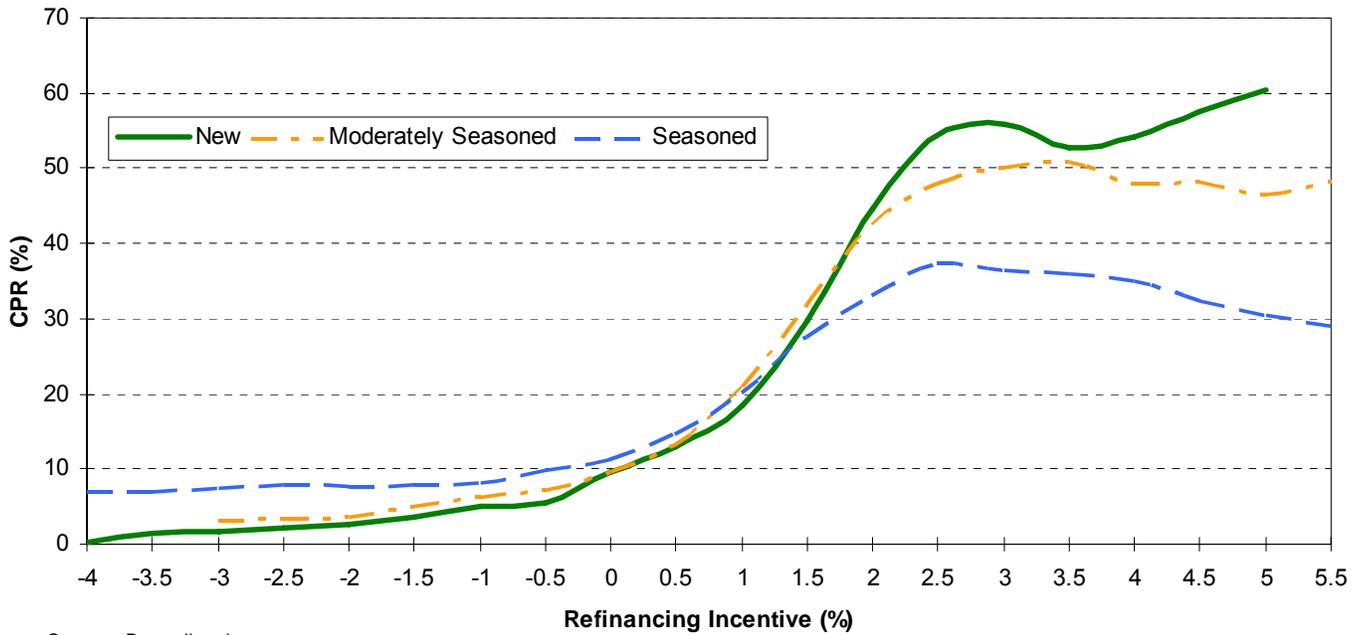
Sources: Beyondbond

Note: Refinancing Incentive is defined as the difference between WAC and 2-month lagged prevailing interest rate.

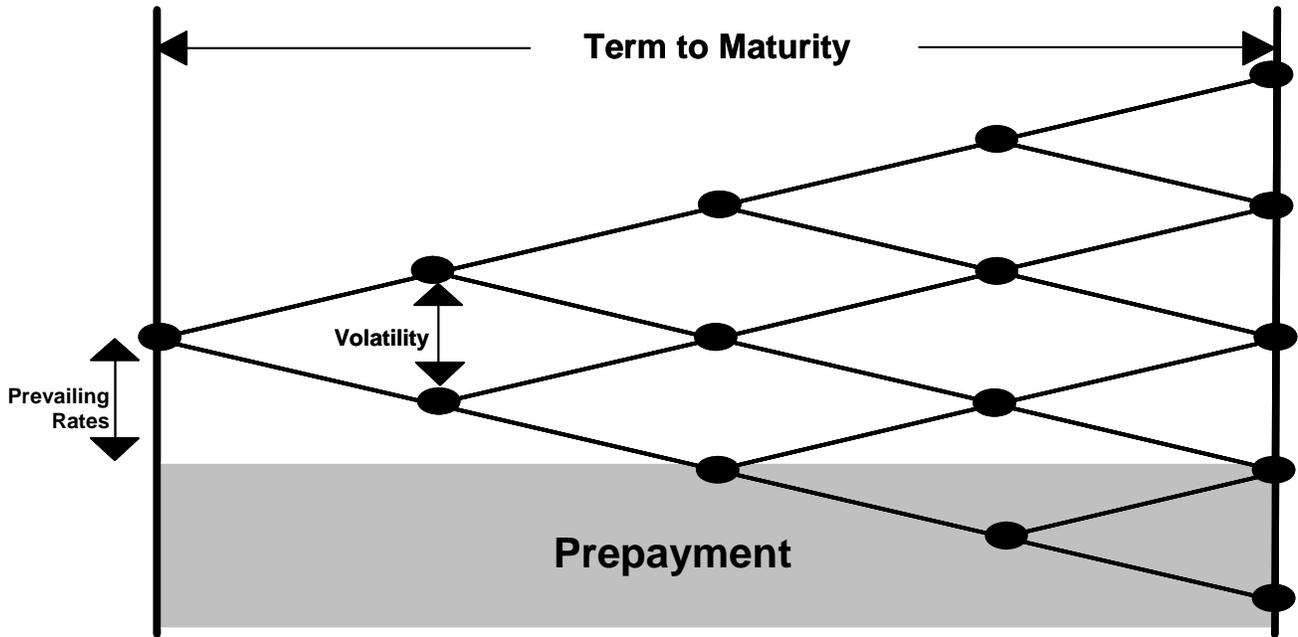**Figure 11.**   **Principal Payment at Different Prepayment Speeds**

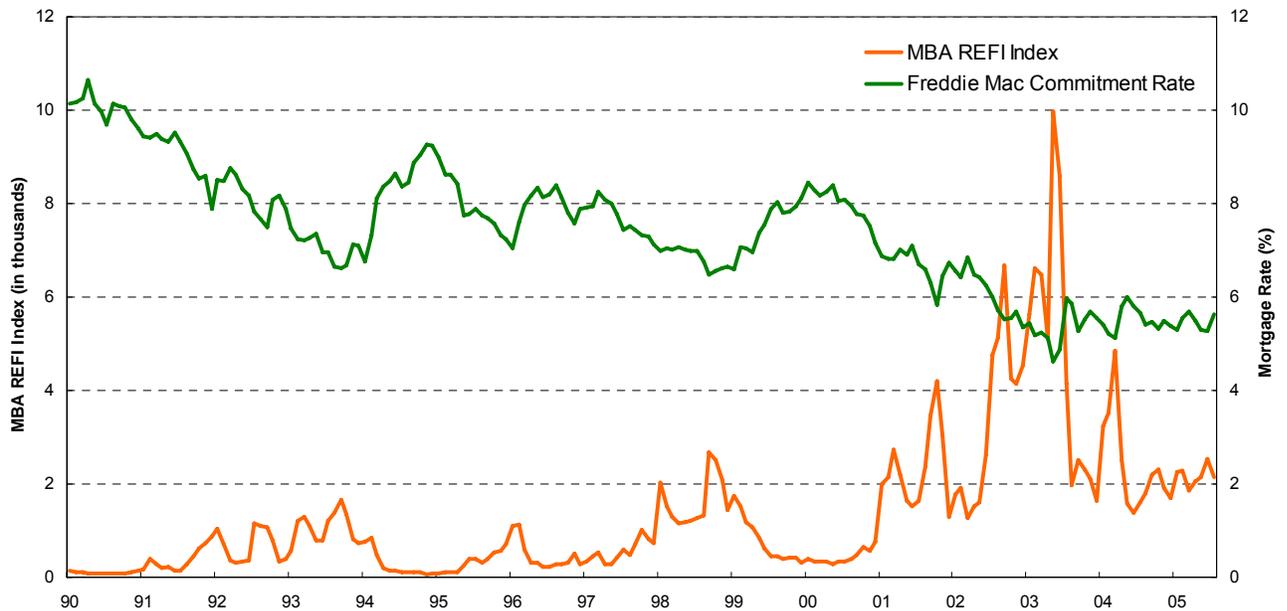Sources: Beyondbond

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Figure 12 .   **Rate Incentive Prepayments**

Sources: Beyondbond

Figure 13.   **MBA REFI Index vs. Freddie Mac Commitment Rate**

Sources: Bloomberg and Beyondbond

Age

Mortgages generally display an age pattern. In Figure 14, the refinancing pattern changes as mortgages become more seasoned. It shifts upwards as the refinancing incentive falls below 1% for both discount and low premium coupons mortgages. For a new home owner, he/she is less likely to relocate during the first few months due to moving costs involved and the time needed to adapt to unfamiliar atmosphere from either social or occupational aspects. In other words, prepayments tend to be very small during the first few months after mortgage origination. Then it rises gradually until it reaches a peak (fully seasoned) and then remains at that level till maturity. The Standard Securities Association (PSA) function which is independent of interest rates has been considered as a canonical age pattern.[3] Figure 14 shows an estimate of the Beyondbond Prepayment Model discount coupon age pattern. This age pattern for discount coupon mortgage is unrelated to interest rates.

Refinance is not influenced by relocation for the premium coupon mortgages as long as a refinancing opportunity exists. Thus the time period to reach peak should be shorter, or not even exist.

Figure 14.   **FNMA 30-Year Age Pattern for Discount Coupon**
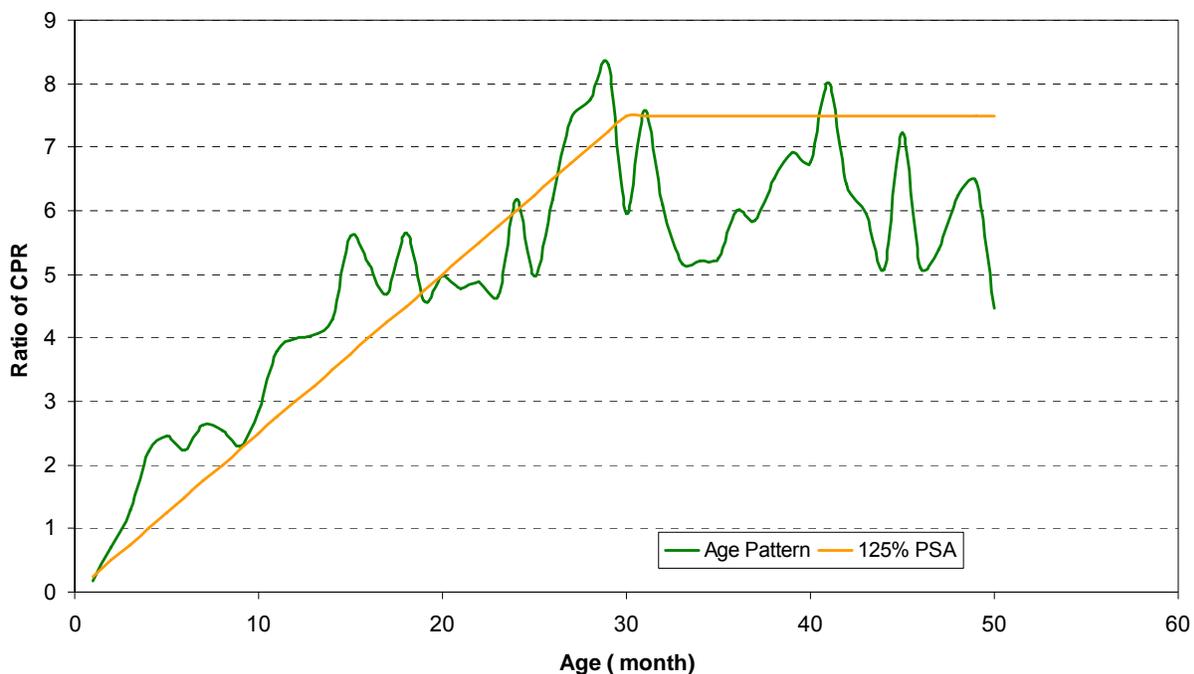
Sources: Bloomberg and Beyondbond

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[3]The PSA speed of 100% starts from 0% per annum Cumulative Prepayment Rate (CPR) at issuance, increases linearly to 6% CPR at month 30, and then remains constant.
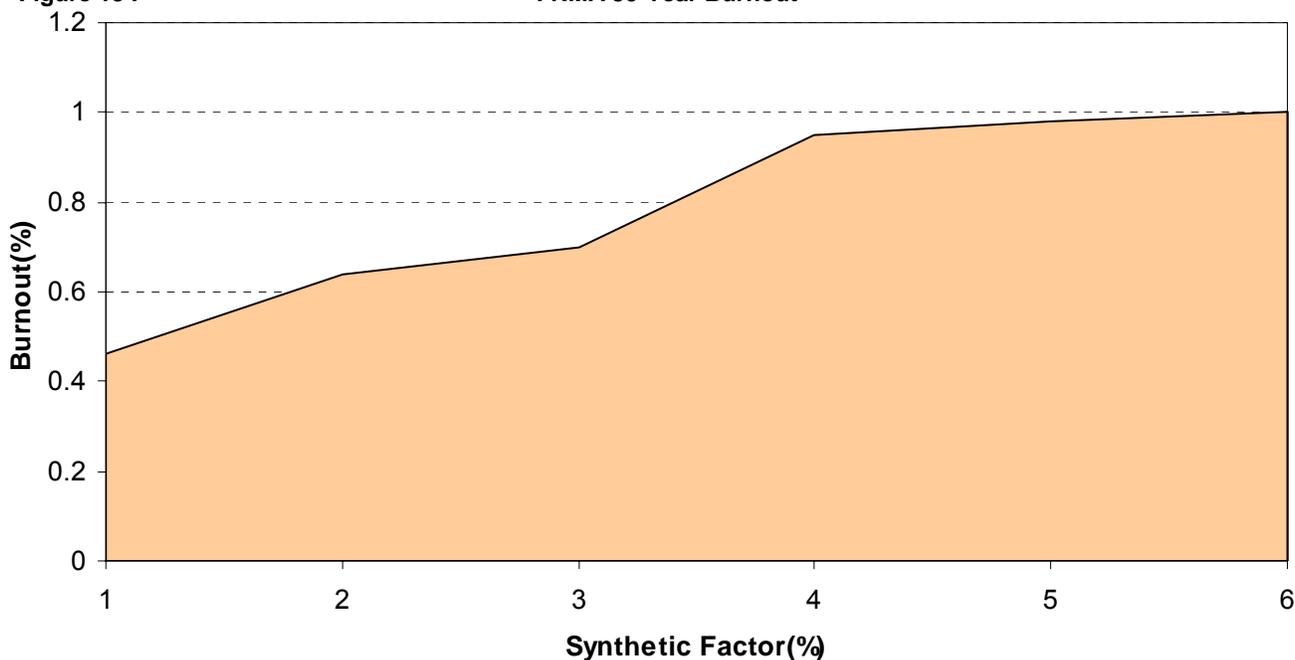
Burnout

While all the mortgagors may have in-the-money prepayment options, they can respond differently to their "favorable" interest rates for various reasons. In addition, loan qualification difficulties, sickness, travel abroad, job loss, relocation, etc., can delay or miss the refinancing opportunity. If the refinance opportunity persists or turns even more attractive, mortgagors, who may have missed the opportunity before, may now decide to refinance at this time. The prepayment rates at this time, however, can be less given the same refinancing incentive because these mortgagors form a less sensitive group. In other words, those who could have refinanced would have refinanced before, so when a subsequent refinancing opportunity comes around, the refinancing rate is not as high as the previous time. This is referred to as a burnout.

The remaining factor, which is the ratio of remaining balance over original balance of a mortgage pool, has been usually considered as a proxy to measure burnout. The remaining factor, however, is pool specific. Also, it excludes both prepaid principals and amortized principals. Burnout, as the decreased part of premium coupon's prepayments which is only related to prepaid principal of in-the-money portion, will be distorted by using the remaining factor. A synthetic factor is thus introduced as a better proxy for measuring burnout in the Beyondbond Prepayment Model.

The synthetic factor is a pseudo-variable through a complicated non-linear function. The function takes the entire pool prepayment projection history since issuance and accumulates only the prepayment due to the exercise of the in-the-money option to form the factor. The usage of the synthetic factor serves four purposes. First, it decreases only as refinance incentives emerge, which is immune from the principal amortization schedule. Second, it checks the whole historical prepayment projections, preventing a specific pool factor distortion. Third, it uses the mortgage's age for the accumulation of the factor so that a seasoned mortgage issued earlier than the securitized date, which produces a higher remaining factor, would be irrelevant to burnout. Finally, it accumulates the excess prepayment due to in-the-money option exercise over the current coupon's prepayments since issuance. Figure 15 shows an estimate of FNMA 30-year burnout versus synthetic factor.
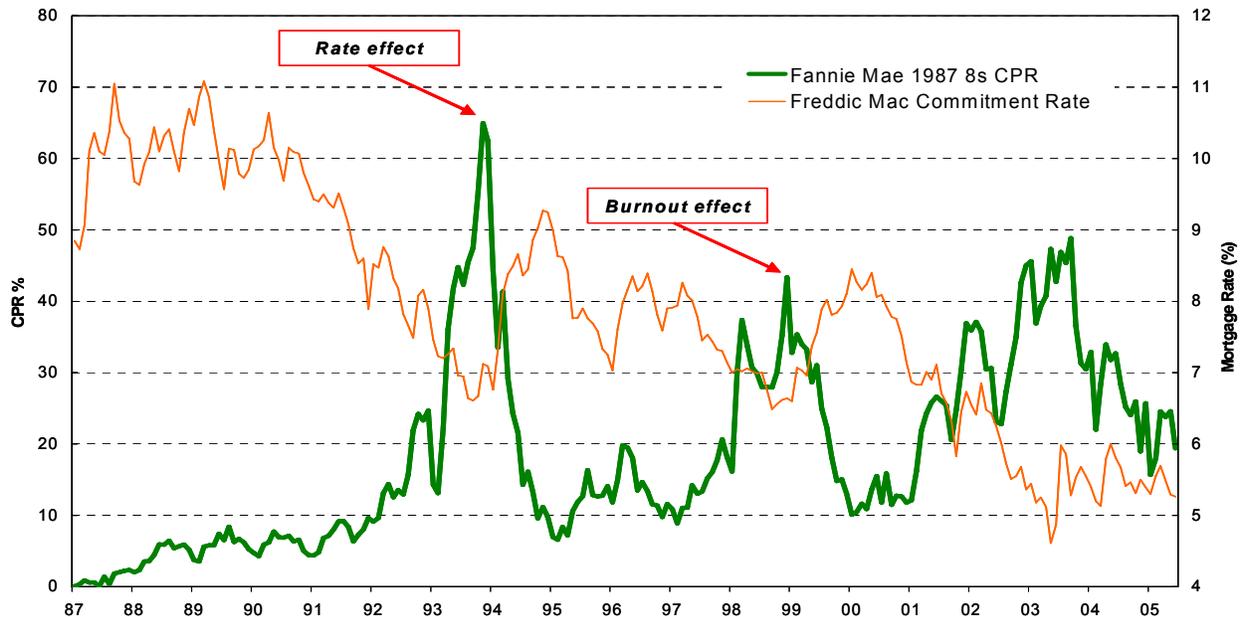
Figure 15 . **FNMA 30-Year Burnout**

Sources: Beyondbond

Figure 16. **Rate and Burnout Effects (Fannie 8s 1987 Issue)**

Sources: Fannie Mae, Freddie Mac, and Beyondbond

Yield Curve

Since late 1991, interest rates have dropped and were at the lowest levels at the end of 2003. Although, faster prepayments were anticipated as a result, the huge volume of refinancing activities through 2003 was greater than most prepayment models' expectations. With such a steep yield curve environment (more than 250 basis points refinance between 2- and 10-year Treasuries over the year of 2003 as shown in Figure 17), the shape of the yield curve has attracted attention as the prepayment component of cross maturities refinancing.

The yield curve factor assumes that mortgagors can refinance across various mortgage maturities if a relative refinancing opportunity exists. For example, the monthly payment for a 10% 30-year mortgage is $877.57. For a 6.62% 15-year mortgage with the same amount of payments, the payment period is shortened by 15 years. As the yield curve turns steeper and steeper, the relative payment advantage of the 30-year over the 15-year mortgages shrinks while the longer payment period disadvantage of the 30-year remains. Similarly, 30-year mortgages can also be refinanced into adjustable rate, 7- or 5- year balloon mortgages. It is difficulty to measure the allocation of prepayments derived from the yield curve factor to other maturity mortgages. However, it is possible to empirically estimate the yield curve portion of prepayments. Since the 30-year mortgage commitment rates, FHCR 30Y, have synchronized with 10-year Treasury yields, T10Y, as shown in Figure 18, the original refinancing incentive combined with the difference between 10- and 5-year Treasuries serves as a proxy for measuring the yield curve portion.
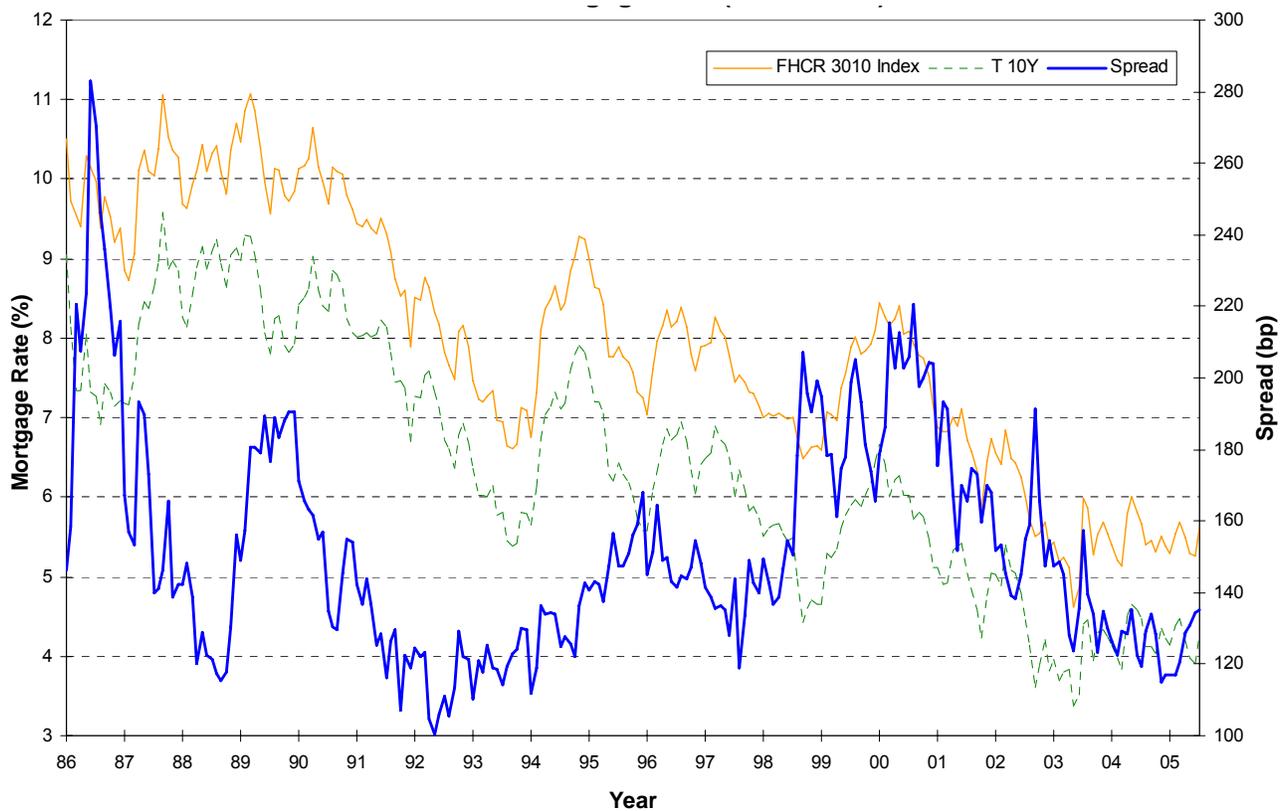
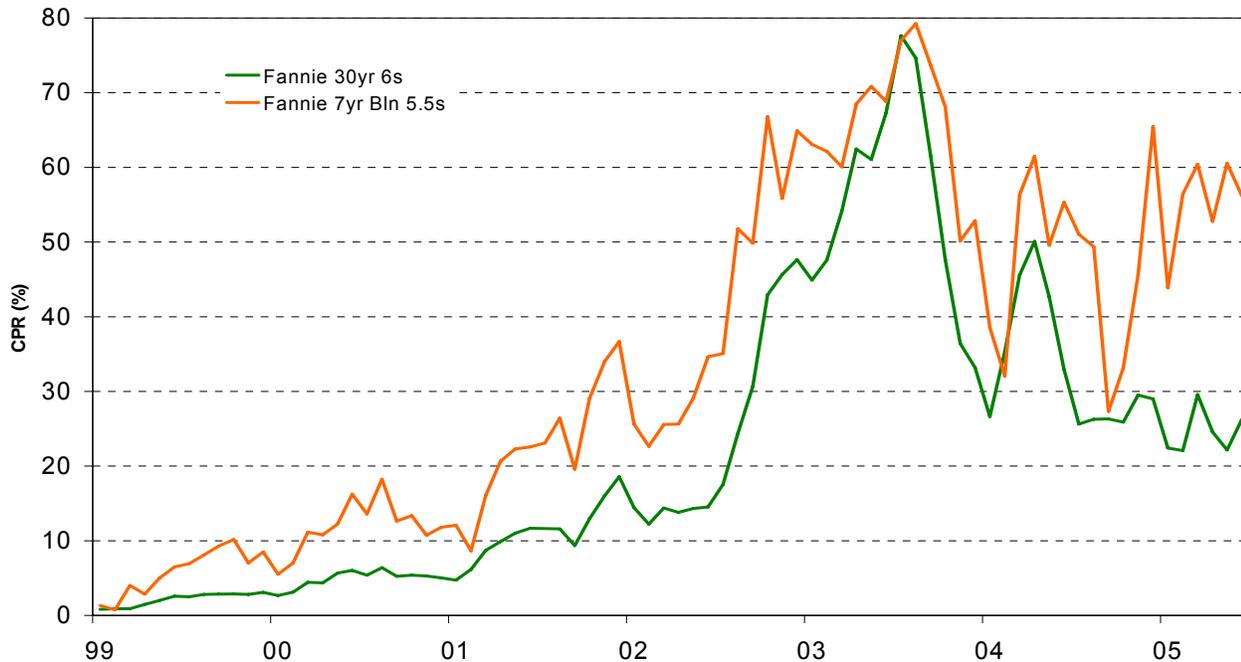**Figure 17.**   **Historical Treasury Yields (2000 – 2005)**
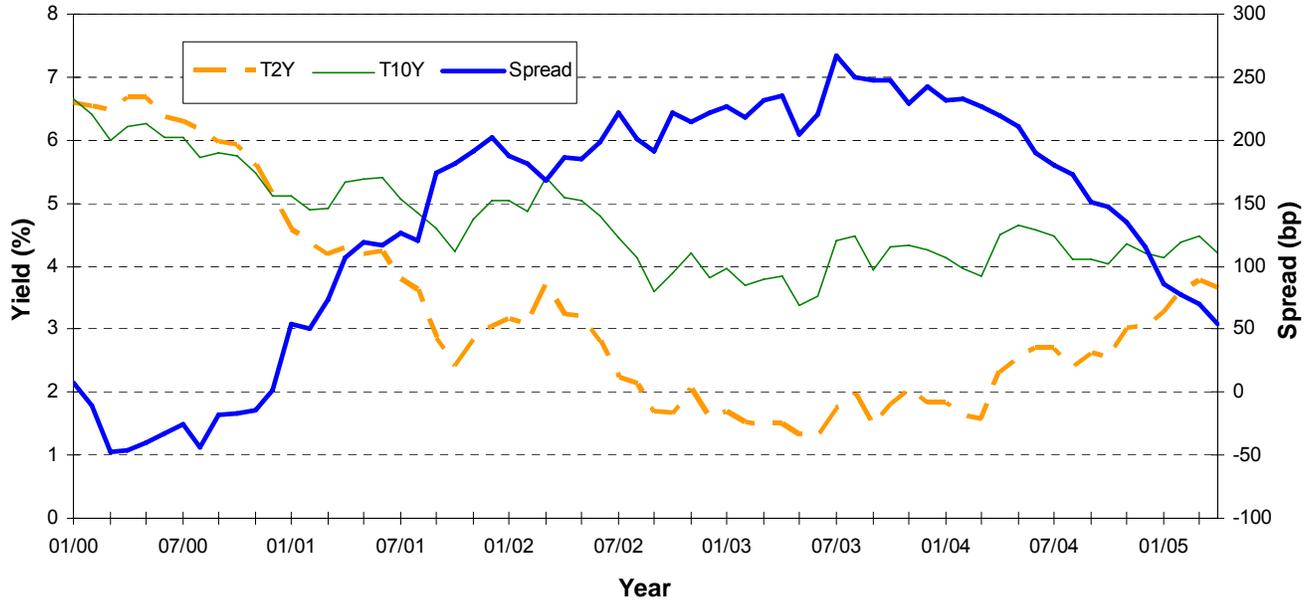
Sources: Bloomberg and Beyondbond

Figure 18.   **Historical Mortgage Rate (1985 -2005)**

Sources: Bloomberg and Beyondbond

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Figure 19.   **Fannie 30 Year 6s vs. 7 Year Balloon 5.5s**

Sources: Bloomberg

Curtailment and Early Payoff

Each month, a payment in the amount greater than the monthly scheduled amount is paid, thereby creating a partial prepayment.

Determinants for Defaults

Loan-to-Value Ration (LTV)
- Original LTV (OLTV)
- Combined LTV (CLTV)

Debt-to-Income Ratio (DTI)

Credit History
- FICO scores

Model Specification

In order to develop a prepayment model, variable selection, model specification, statistical estimation, and predictability are usually performed. A priori is essential for the model specification. A prepayment model with same variables yet different specification may derive different results. The statistical estimation and goodness of fit also affect the criteria for model selection.

The Beyondbond Prepayment Model which employs economic variables discussed before includes these prepayment components:

‣ Issuer Agency
‣ Amortization Type
‣ Housing Turnover Rate
‣ Cash-out
‣ Equity
‣ Credit
‣ Refinance
‣ Age
‣ Seasonality
‣ Burnout
‣ Yield Curve
‣ Issue Year Component
‣ Financing Expense Component

Essentially, agency and amortization types are considered as demographic variables, and their prepayments are modeled separately within each category. The model is multiplicative among Refinance (**SP**), Age (**AG**), Seasonality (**SN**), Burnout (**BF**), Yield Curve (**YC**), Issue Year Component (**IY**), and Financing Expense component (**FE**). The model Is additive in the logarithm format while estimating to ensure a non-negative prepayments. For each factor, a spine function is applied and a cubic spine is used for certain factors to ensure a higher degree of continuity and smoothness. This design provides the model with an elegant but flexible mathematical specification.

A Multiplicative Spline Model

Let CPR be the annualized Constant Prepayment Rate, our prepayment model is the multiplicative product of following seven components, Refinance, Age, Seasonality, Burnout, Yield Curve, Issue Year Component, and Financing Expense Component and is formulated as followings:[4]

$$[CPR] \ = \ [SP] \ \cdot \ [AG] \ \cdot \ [SN] \ \cdot \ [BF] \ \cdot \ [YC] \ \cdot \ [IY] \ \cdot \ [FE]$$

Each component or factor forms a separate spline function. The Refinance factor with a cubic spline functional form is the primary prepayment driver while Age, Seasonality, Burnout, Yield Curve, Issue Year Component, and Financing Expense Component are constructed as secondary to the Refinance factor.

Figure 20 provides an example of estimated FNMA 30-year MBS refinancing function on a PSA basis. The PSA format is used as a more intuitive prepayment measuring unit. The corresponding SMM can be easily converted via a fully seasoned PSA schedule. A 150% PSA, for instance, should be equal to 0.78% SMM. A 375% PSA, however, should be equal to 1.05% plus 0.92% SMM instead of 2.10% because current coupon PSA is at 370% level.

Figure 20.   **Estimated Refinancing Function**

FNMA 30-Year Refinancing Incentive
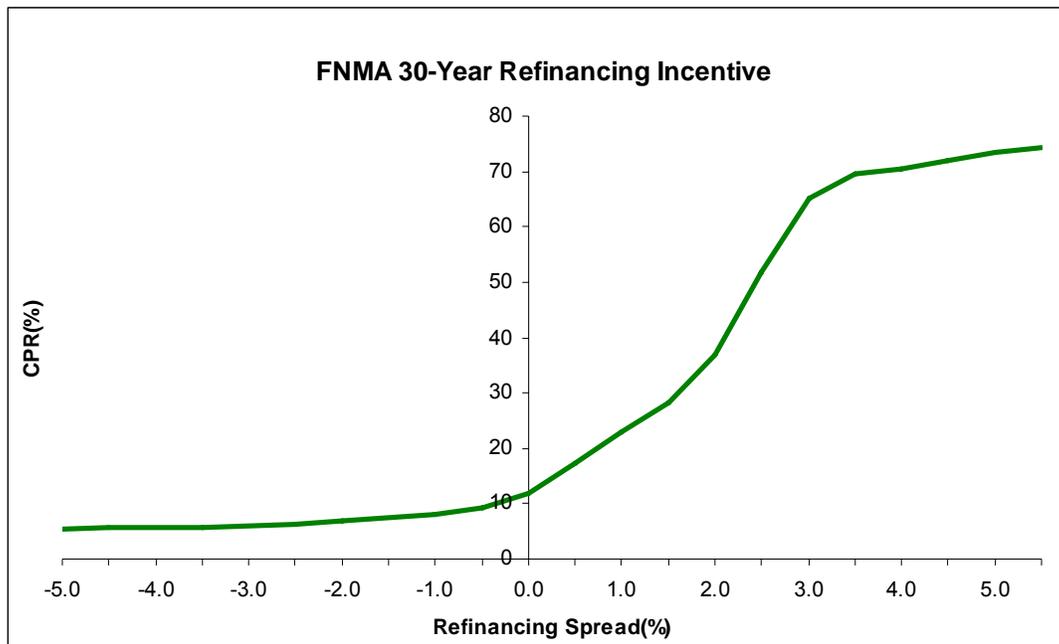
Note: The PSA numbers are assumed at fully seasoned level.

Sources: Beyondbond

Numerical Illustration

A numerical example demonstrates the model in use. Suppose a FNMA 30-year mortgage of 8% WAC actually prepays at 584% PSA at month 5 since issuance, an equivalence of 0.5% SMM. Assume that the burnout and seasonal factors are both equal to one, and the curve factor is equal to zero for all months. If the prevailing refinancing mortgage rate is equal to 7%, the refinancing difference will be equal to 1%. Figure 21 shows that Rate_I at the current coupon level is equal to 175% PSA (equivalent to 0.92% SMM). Let Age_I be the same as the PSA age function which produces a 16.67% level at month 5. A 0.15% SMM (0.92% times 16.67%) is calculated as the discount part. The remaining 0.35% SMM (0.5% minus 0.15%) should be contributed by the premium part. Since Rate_R factor at 1% refinancing refinance level is 85% PSA (260 minus 175) which is equivalent to 0.44% SMM, Age_R should be at 79.5% level (0.35% divided by 0.44%) on the age of 5 month.

To show the burnout, assume the mortgage age to be 85 months with both Age_I and Age_R equal to 1, we have 0.92% and 0.44% SMM's for the discount and premium parts, respectively. If the synthetic factor is equal to 55% which, according to Figure 4, produces a burnout factor at 97% level, then SMM will be 1.219% (0.92% plus premium part, 0.299, which is equal to 0.44% times 68%).

Since the seasonal factor are denoted as monthly coefficients, continuing with the previous case, SMM will be 1.618% which is comprised of 1.334%, 0.92% multiplied by 1.45 for the discount part, and 0.284%, 0.299% multiplied by 0.95 for the premium part.

The "curve" part is based on 10- and 5-year Treasuries difference. Assume the difference is 150 basis points. Since the original refinancing difference is 1%, the new refinancing difference will be 2.5%, corresponding to 950% PSA based on the Figure 20. Applying the same refinancing function with age, seasonal, burnout factors, we can calculate a new SMM, called P15, equal to 4.616% to differentiate from old SMM, called P30, which is equal to 1.618. A coefficient, α, is created to adjust the difference between P15 and P30. The curve part will have impact of 1.049% as α equal to 0.35. Thus, the overall prepayment rate will be equal to 2.667%, 1.618% plus 1.049%. Note that only the positive part of P15 over P30 is taken into account, which means that any flat or inverted yield curve will not generate an impact. If α is set to be zero, even with positive sloped curve, no curve impact will emerge. Figure 21 summarizes the illustration.

Figure 21.

Mortgage Characteristics:	FNMA30-Year, WAC=8%, Age=85			
Interest Rate Environment:	FRM30Y=7%, T10Y-T5Y=1.5%			
Coefficients:	Seasonal_I_5=1.45, Seasonal_R_5=0.95, α =0.35			
Discount Part	Age_I	Seasonal_I	Rate_I	
1.334%	1.00	1.45	0.92	
Premium Part	Age_R	Seasonal_R	Rate_R	Burnout
0.284%	1.00	0.95	0.44%	68%
Curve Part	α	P15	P30	
1.049%	0.35	4.616	1.618	

IV. EMPIRICAL STUDY

It is important to have model empirically tested in order to obtain reasonable coefficients. In spite of demographic variabilities, for each observation date, there are several SMM's for various coupons and issue-years. This type of cross-section-time-series data in which the same cross section is sampled over time is called panel data. The maximum likelihood (ML) estimation with Gaussian assumption is utilized to obtain the coefficients. The marginal likelihood function is based on the distribution of SMMt,I conditional on Xt, where Xt is the information set up to date t. A general form of the logarithmic likelihood function which accumulates marginal likelihood function is given as follows:

$$\ln L(\theta \mid X_T) = 2 \cdot \sum_{t=1}^{T} \sum_{i=1}^{i_t} \ln(w_{t,i}/\sigma_t) + \left(\sum_{t=1}^{T} \sum_{i=1}^{i_t} \{[(SMM_{t,i} - f(X_{t,i};\theta)) \cdot w_{t,i}/\sigma_t]^2\} \right)$$

Where θ is a vector of parameters; the indices t, i represents time and mortgage type (such as net coupon and issue-year) correspondingly; wt, i denotes the weights for each observation. Heteroskedascity (non-constant variance of the error term) and local maximum problems, however, are considered. The goodness of fit and out-of-sample predictability serves as the criteria for our fitting process.

Assuming the variance over the whole data set, i.e., (σt / wt,i) to be constant may be too strong for a panel data. Different method to weigh data, however, can result in substantially different estimated coefficients. With the simple assumption of σt to be constant, the model weights, wt,i's, are based on their outstanding pool balances which presumably give more weights on recent data, especially for the current coupon mortgages. These weights provide greater accuracy for larger pools and implicitly emphasize greater validity for recent current coupon mortgages as the MBS market has grown for the last several years. It may not be the best proxy to justify heteroskedascity, but it provides for a better sense of fitting in practice. The complexity of non-linearity in parameter space may cause local maximum convergence while employing MLE. An improper assumption on initial values can lead the model to converge to a non-intuitive set of coefficients. A carefully designed initial values set with an economic intuition is important prior to estimation. The R2 statistics of 94.7% and 90.2% for GNMA and FNMA 30-year MBS, respectively, are fairly high although they are not very meaningful due to the heteroskedastic adjustment.

Empirical Results

Since the ML estimation minimizes sum of all squared prepayment error terms, it does not necessarily satisfy the fitting purposes for each coupon. Figure 21 gives a summary of fitted versus actual values for each net coupon for the month of September, 2005. The fit for the seasoned mortgages is quite good. The new products, however, need more tuning. Figure 22 shows the graphics of fitted and actual CPR's against incentive refinances. Burnout seems not to be sufficient enough to drag prepayments down as refinancing difference becomes greater than 2%. Figure 23 gives a time series fitted versus actual plots of FNMA 30-Year 7% coupon.

Both in-sample fittings and out-of-sample predictions capture the actual prepayment trend, although the predictions seem uniformly underestimated which suggests a refit with recent data. A fine tuning of coefficients would be necessary to fulfill different fitting criteria. The impacts on SMM through each prepayment factor were discussed in the previous numerical illustration.

Figure 21.   Prepayments in PSA for FNMA 30-Year for the month of September, 2005
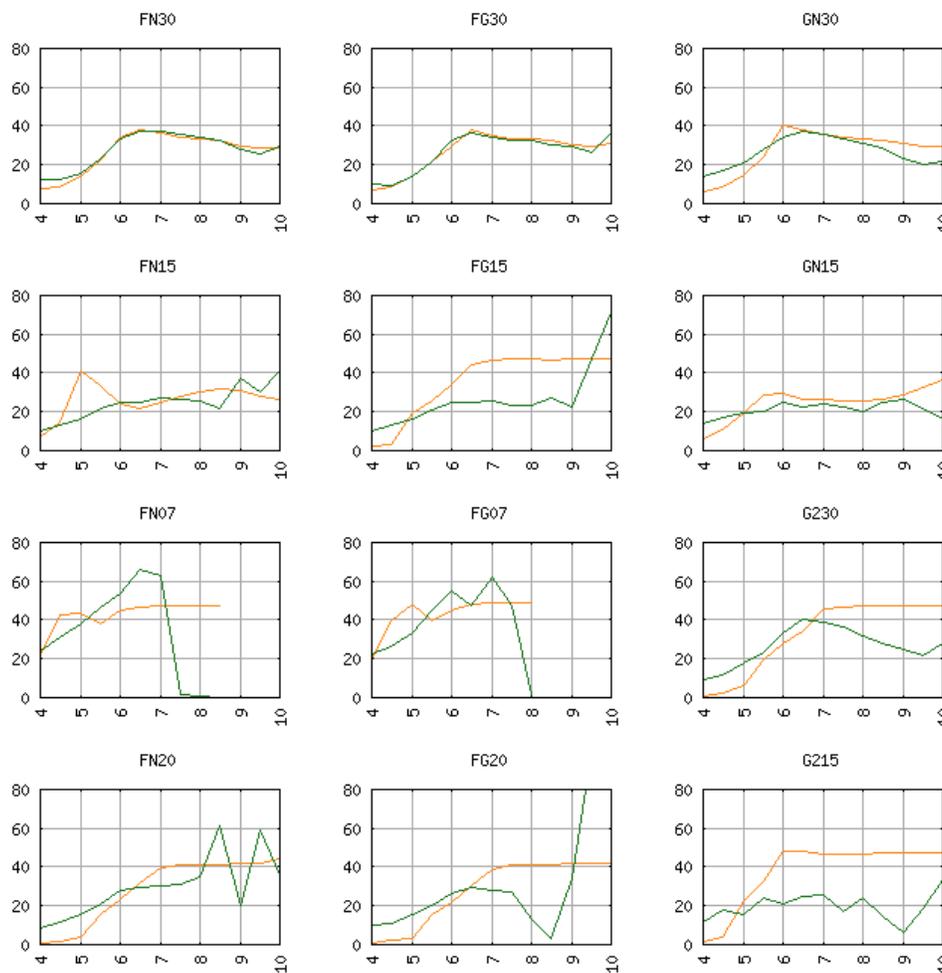
Note: The orange line describes the Beyondbond Prepayment, and the green line describes existing prepayment data.

Figure 22.
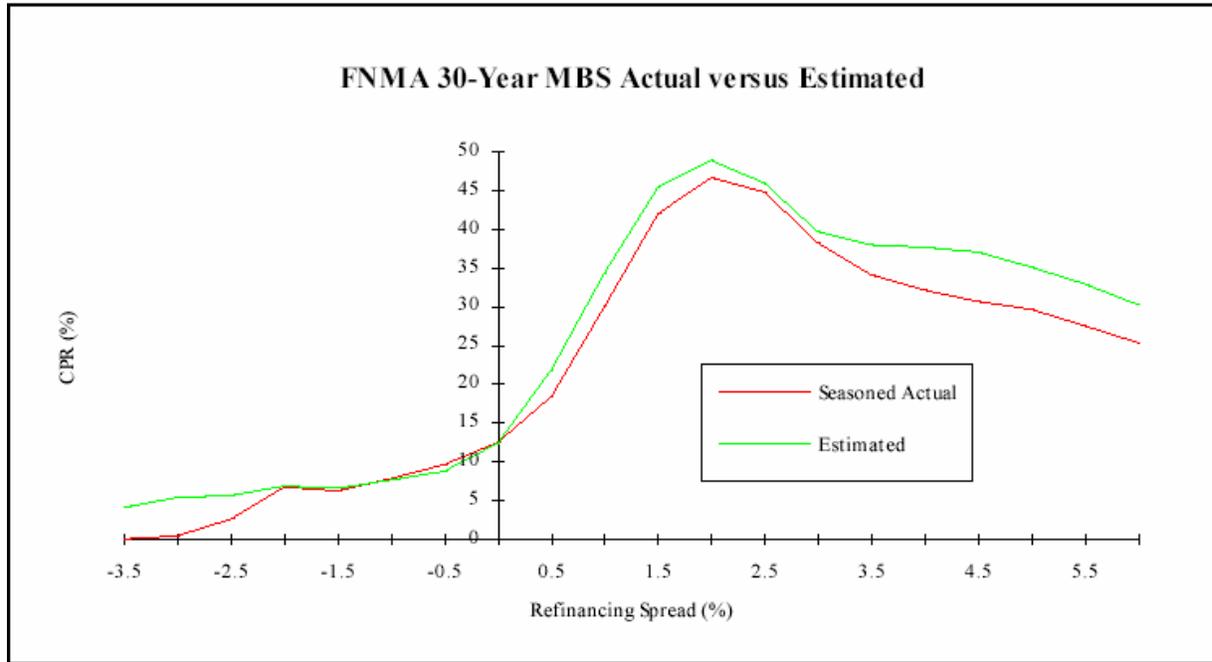
Figure 23.
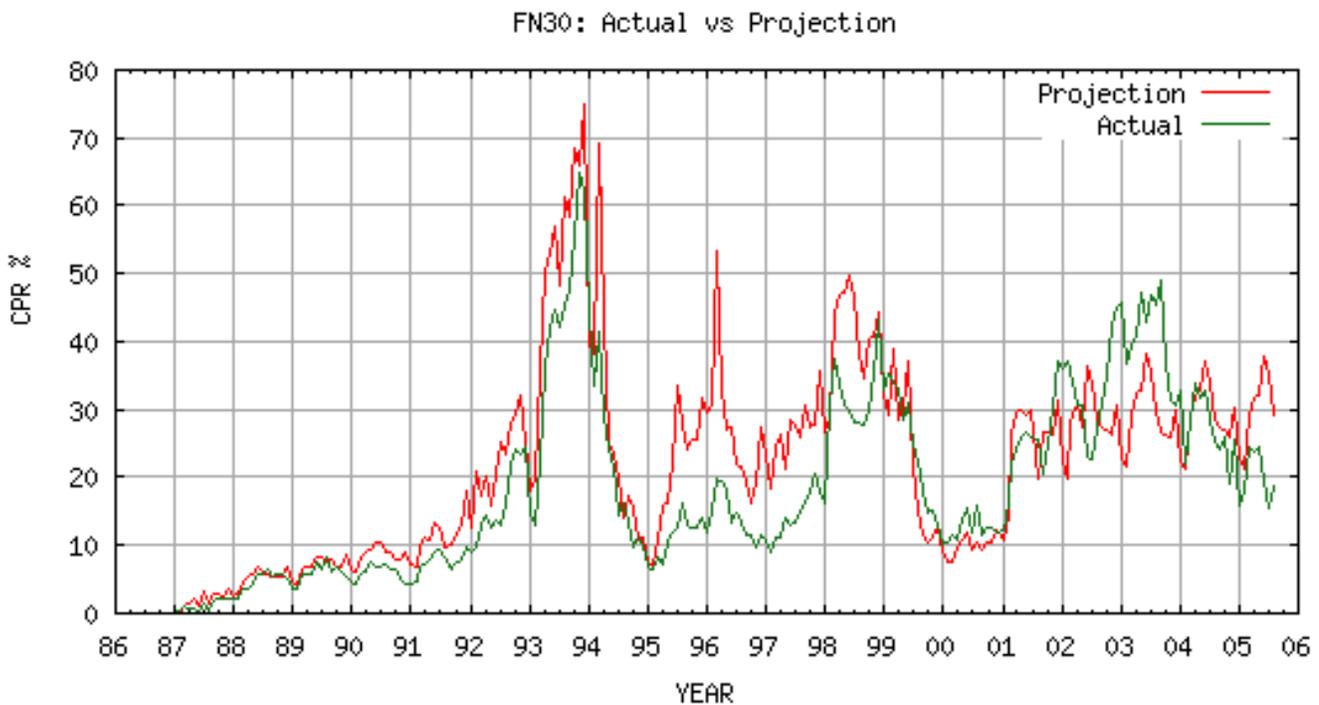
Applications

A prepayment surface based on the ML estimates of FNMA 30-year data and current yield curve environment is drawn in Figure 24. This surface has been commonly applied to project vector, short- and long-term prepayments in scenario analysis. In addition, by linking to the term structure model, one can dynamically project prepayment rates to correspond to a specific interest rate path and apply them to various MBS valuation models. Options adjusted refinance application is a common example for stochastic analysis. The follow-up step after creating a prepayment model then is to construct MBS valuation tools.

Figure 24.   **Prepayments Model Surface without Seasonality**
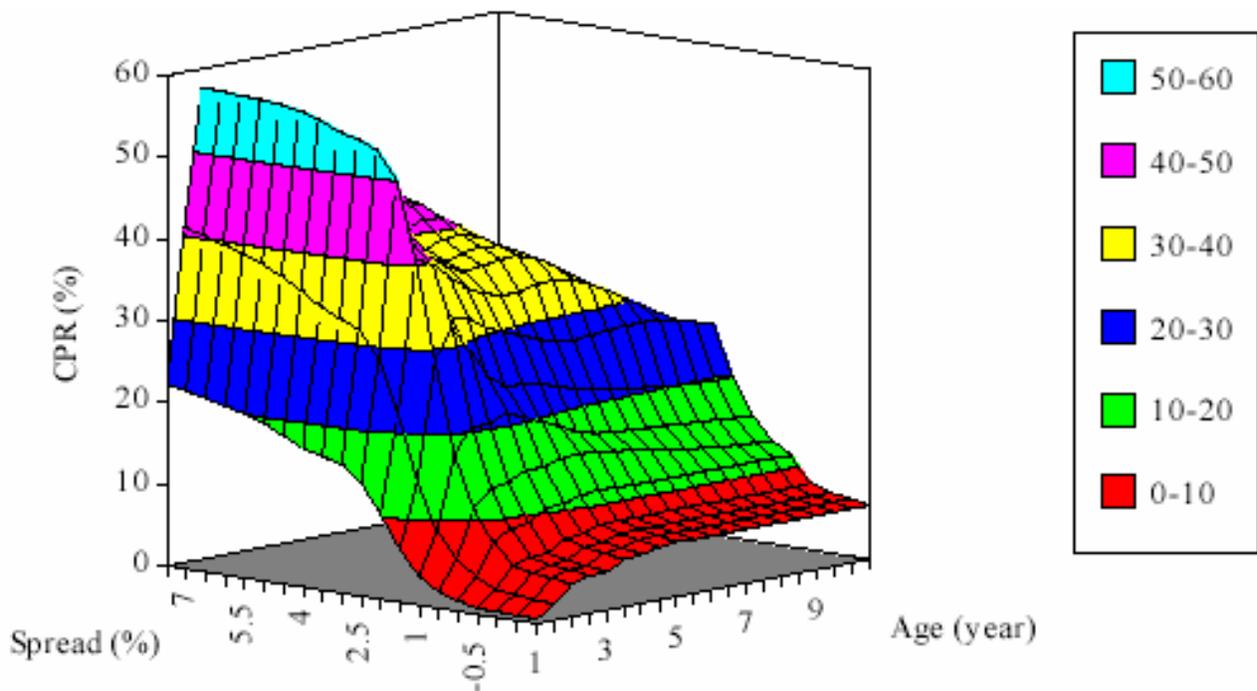
CONCLUSION

Starting with a fundamentally economic theory to analyze mortgagors' prepayment behaviors, the framework for the Beyondbond Prepayment Model was built and then empirically tested. When linked to the term structure of interest rates, the model can be used in a variety of MBS applications such as scenario analysis, total rate of return, and options adjusted refinance.

The model specifies a multiplicative spline to filter various prepayment factors. A cubic spline functional form is employed across the rate, age, and burnout factors. The innovative use of the synthetic factor, which only accumulates in-the-money part of prepayment rates, captures the burnout effect. The yield curve factor, which exploits the information of 5- and 10-year Treasuries, enables the model to measure the relative opportunity for refining into different maturities. The empirical results of in-sample fitting and out-of-sample forecasting support the model specification. Nevertheless, fine tuning the coefficients to fulfill different fitting purpose can be useful.

Modeling prepayments can be a never ceaseless work since more data and newer and more innovative mortgage instruments may generate additional information about a mortgagor's prepayment decision process. Periodical updating of the coefficients would be necessary as a consequence.

APPENDIX

I. Model Specification

The model per se employs cubic spline functions over a variety of variables. Let Single Monthly Mortality (SMM) be formulated as follows:

$$[CPR] = [SP] \cdot [AG] \cdot [SN] \cdot [BF] \cdot [YC] \cdot [FE] \cdot [IY]$$

where
BF: Burnout factor
SP: Refinance factor, prepayments from the cumulated in-the-money call options.
AG: Age factor from premium coupon part
SN: Seasonal factor from premium coupon part
YC: Yield curve factor, prepayments due to cross maturity incentive
FE: Financing Expense Component.
IY: Issue Year Component

Age Factor:
AG = AG(k) is a spline function of age k.

Seasonal Factor:
SN = SN_j is a constant coefficient on the j_th month of the year

Refinance Factor:
SP = SP(x) is a cubic spline function of refinance x.

where
x is the ratio refinance of the underlying mortgage rate over a lagged prevailing 30-year mortgage rate multiplied by 10.

Yield Curve Factor:
YC = YC(xc) is a spline function of refinance xc.

where
xc is the refinance of WAC over two months lagged prevailing 15-year mortgage rate the 15-year mortgage rate takes [x + (tsy10-tsy5)] as the proxy.

Burnout Factor:
B = $B(S_t)$ is a spline function of synthetic factor S_t.

where

$$S_t = \prod_{i=1}^{t} Max(1-SMM_i, 0)$$

$$S_0 = 1$$

II: DEFINITION OF THE CUBIC SPLINE FUNCTION

A function f(x) is a cubic spline function of x if it satisfies the follows:

$$f(x) = \sum_{i=1}^{4} A_{ij} Max(x - d_j, 1)^i$$

$$\text{subject to} \quad \begin{aligned} f(d_j) &= f(d_{j+1}) \\ f'(d_j) &= f'(d_{j+1}) \\ f''(d_j) &= f''(d_{j+1}) \end{aligned}$$

$$\forall j = 1, \ldots, m$$

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NEW YORK, USA
Headquarters
45 Broadway
9th Floor
New York, NY10006
646-313-3330

SHANGHAI, CHINA
No. 888 Yi-Shan Road
Cao He Jing Science Park
Shanghai, China

TAIPEI, TAIWAN
No. 246 Cheng-De Road
Sec. 3, 6th Floor
Taipei, Taiwan

LOS ANGELES, USA
LA Office
161 Mercury Circle
Pomona, CA 91768